# Pre-class Warm-up!!!

Do you remember last time we did a lot with the most × [34]?

True or False?

The matrix  $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$  has 3 eigenvectors, no two of which are scalar multiples of each other.

a. True

b. False

It has up to scalar multiple any the e-vectors (6], [] with e-values

2 and 3,

It has eigenvectors [1], [1] with

eigenvalues 7, -1

## 6.2 Diagonalization of matrices

New vocabulary:

• diagonalize, diagonalizable, similar

We learn:

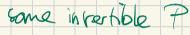
- the connection between eigenvalues, eigenvectors and diagonalization
- how to diagonalize a matrix (when it is diagonalizable).
- Some theorems: a criterion for diagonalizability; independence of eigenvectors when eigenvalues are distinct; distinct eigenvalues implies diagonalizable.

What we don't really learn:

• why we would want to diagonalize matrices

Definition. Square matrices A and B are similar if there is an invertible matrix P = 0 that  $B = P^{-1}AP$ 

A square matrix A is diagonalizable if If IC



Definition. Square matrices A and B are similar if there is an invertible  $n \ge n$  matrix P so that  $P^{-1}AP = B$ the e-vectors of A and the

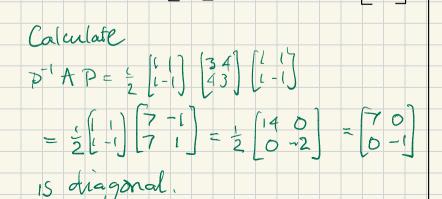
A square matrix A is diagonalizable if it is similar to a diagonal matrix.

diagonal entres of [3 -1] are the

corresponding e-values

Example:  $A = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$ 

Try the matrix  $P = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$  so  $P \land \{-1\} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ 



Theorem. Let A, P, D be n x n matrices with P invertible and D diagonal.

Then  $P^{-1}AP = D$  if and only if the columns of P are eigenvectors for A with eigenvalues the diagonal entries in D.

Proof.  $P^{-1}AP = D \iff AP = PD$ Write the columns of Pas Vizing vn  $P = \left[ v_1 | v_2 | \dots | v_n \right], \text{ Write } D = \left[ \begin{array}{c} \lambda_1 \\ 0 \\ 0 \end{array} \right]$ AP=PD means: for each i AV: = it is of AP ~  $\lambda_i v_i = i^{th}$  column of PD  $e_{g}[43][1] = [7-1] [1] [7] = [7-1] [1] [7] = [7-1] [7-1] [7-1] = [7-1] [7-1] = [7-$ 

€) for each i, Avi = λ. Vi

Es for each i, vi is an e-vectorsf A with e-value  $\lambda_i$ 

Theorem 1. An n x n matrix A is diagonalizable if and only if A has n linearly independent eigenvectors.

## e-vectors

E) there is an invertible matrix P with columns that are e-vectors of A

E) there is invertible Phinth P'AP=D is diagonal,

<>> A le chaophalizable.

Example. The matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable.	Like 6.2 questions 1-28 Find whether or not the following matrices are diagonalizable. If so, find P so that
Proof. We show A does not have 2 independent e-vectors. Find e-values:	$P^{-1}AP = D \text{ is diagonal.}$ $1. A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}  2. A = \begin{bmatrix} -5 & -12 \\ 3 & 7 \end{bmatrix}$
Characteristic poly: det [1-2]	Solution: Charpoly $(I-\lambda)(3-\lambda)$ . Two e-values $I:$ find nullspace of $A - I = \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}$ .
$=(1-\lambda)^2$ Roots: 1 (twice).	It have basis (0] It have basis (0]
TO Find e-vectors: find Null (+->I) = Null [00]	It have basis [0] 3. Null (-22) have basis [1] (10]
1 free variable. [b] is a basis far	Take $P = \begin{bmatrix} 1 & i \\ 0 & i \end{bmatrix}$ , Then $\overline{P} A P = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$
the nullspace, There is one e-vector	
up to scalar multiple. A is not diagonalide Also [!], [2], [2], [2] are not diagonalizable -	Image: Strain
NUI OVINGONALIZABLE -	

#### Like 6.2 questions 1-28

Find whether or not the following matrices are

diagonalizable. If so, find P so that

 $P^{-1}AP = D$  is diagonal.

1. 
$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$
 2.  $A = \begin{bmatrix} -5 & -12 \\ 3 & 7 \end{bmatrix}$ 

Two more matrices:

Solution 2. Char. 
$$puly = det \begin{bmatrix} -5-x -12 \\ 3 & 7-x \end{bmatrix}$$

$$= \lambda^{2} + 5\lambda - 7\lambda - 35 + 36 = \lambda^{2} - 2\lambda + 1 = (\lambda - 1)$$

$$\lambda = l$$
 is the only root, twice.

Eche (on form [0 5]. Basis for nullspace [2]. There is only on e-vector up to scalar multiple. A is not diagonalizable b. c.

Question: How many distinct eigenvalues does matrix 4. have? If you get to it: is it diagonalizable?

a. 0

2

Theorem 2. If A has eigenvectors  $v_1, \ldots v_k$  We get a shorter relation if r > 1associated to distinct eigenvalues, then  $v_1, \ldots, v_k$  are independent. This is a contradiction. The

Proof. Let the e-values be  $\lambda_1, \ldots, \lambda_k$   $V_1, \ldots, V_k$  are independent.

 $AV_i = \lambda_i v_i$  for each i,

Suppose vis., ve were dependent

Let  $c_1v_1 + \dots + c_kv_k = 0$  be a non-

2000 dependence relation. Rearder the

Vi and let c, v, +...+ Crvr be a shortest

such relation, Apply the matrix A-DrI.

 $(A - \lambda_r I) v_i = A v_i - \lambda_r v_i = \lambda_i v_i - \lambda_r v_i$ 

= 0 i f i = r $\pm 0 f i t + r,$  Theorem 3. If the n x n matrix A has n distinct eigenvalues, it is diagonalizable.

### Page 354 Question 32

Show that if n x n matrices A and B are similar, then they have the same characteristic equation, and therefore have the same eigenvalues. Page 354 Question 29

Prove: if the matrices A and B are similar and the matrices B and C are similar, then the matrices A and C are similar.